THE ECONOMIC COSTS AND BENEFITS OF AN INTERNATIONAL GRAIN RESERVE PROGRAM WITH AND WITHOUT IMPROVED (LANDSAT) CROP INFORMATION:

A Case Study Based On The ECON Integrated Model

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FINAL

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NOTE OF TRANSMITTAL

This final report is submitted to the National Space and Aeronautics Administration, Office of Applications, in fulfillment of Task No. 9, Contract NASW-3047. It is in two parts: (1) Final Technical Report, and (2) Summary and Overview. In addition, two separately bound Appendices (77-294-1A and 77-294-1B) documenting the computer work on the contract have been sent by ECON to the Technical Officer monitoring this task, Mr. S. Ahmed Meer at Goddard Space Flight Center.

ECON acknowledges the efforts of Philip Abram and David Lawson in using the ECON Integrated Model to analyze the costs and benefits of an International Grain Reserve with and without LANDSAT.

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1. BACKGROUND AND MODELING CONSIDERATIONS

1.1 Introduction

Numerous research studies have been conducted in recent years on the economic effects of grain reserve stockpiling by the United States government, many of them government-sponsored [1,2,3,4,5,6,7,8,9]. Some of these studies have been collected into one volume by Eaton and Steele of USDA/ERS in Reference 1. The essential economic goal of most grain reserve policies is price stabilization. A secondary goal is to provide food aid when there are severe food shortages. Price stability on domestic markets has been shown theoretically to provide a net economic benefit to the home society, but there is controversy over the more complicated case of a commodity which is involved in international trade, over the distribution of benefits and over the mechanisms for achieving price stability. Grain prices are highly variable (see Figure 1.1) due mainly to natural variability in growing conditions which affects production. Reserve acquisitions in times of surplus put a floor on grain prices, helping to maintain producers' income. Release from the reserve stockpile at times of shortage puts a price ceiling on grains, helping consumers. Methods of achieving price stability include reserve stockpiling of grains by government, acreage controls, loans to farmers tied to production restrictions, incentives and subsidies for private stockpiling, import/export controls and various forms of direct price support and control policy. At the present time, several factors tend to make a combination of government grain

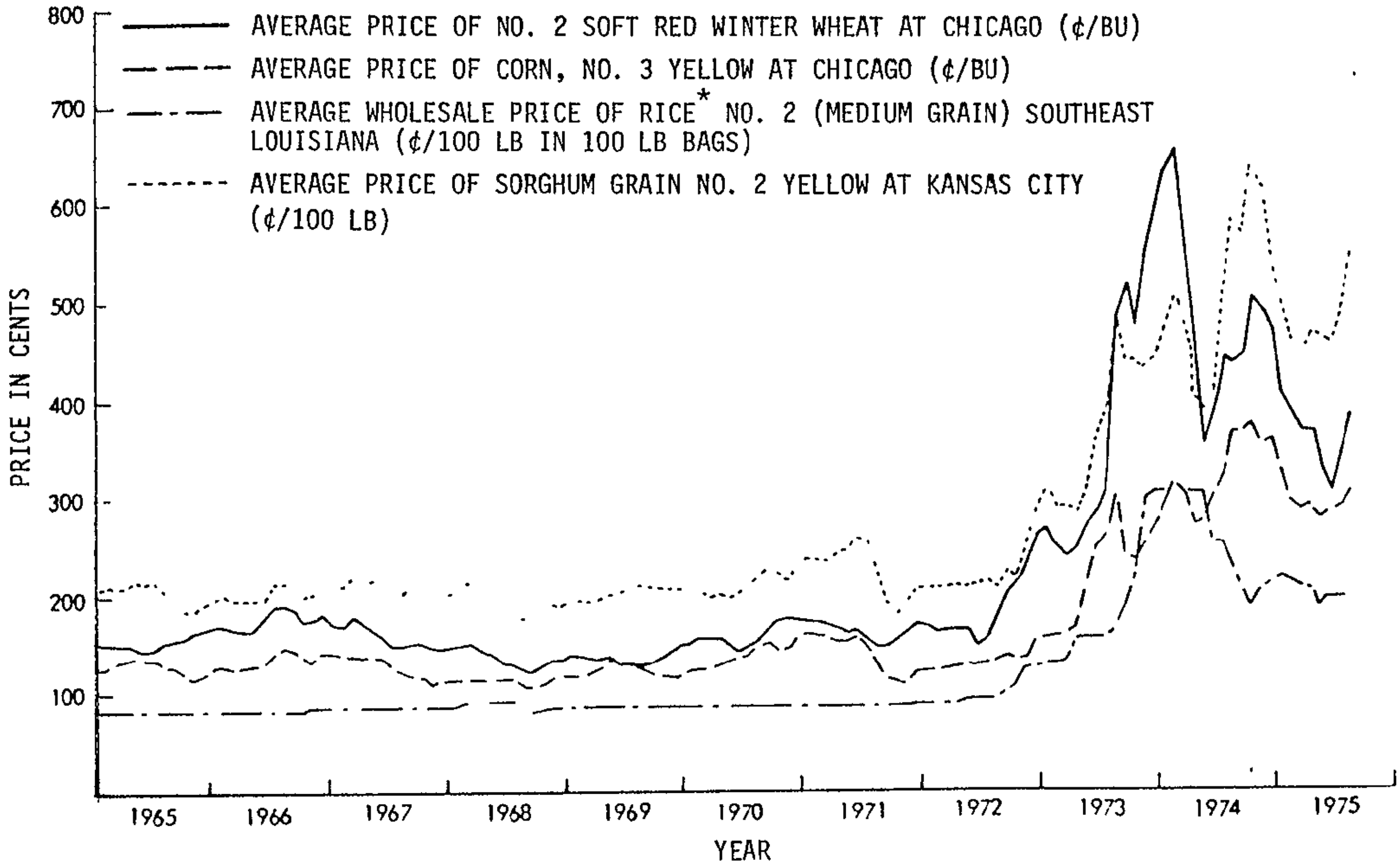


FIGURE 1.1 SELECTED GRAIN PRICES, U.S. WHOLESALE MARKETS (MONTHLY WEIGHTED AVERAGES OF REPORTED DAILY CASH SALES) (SOURCE: AGRICULTURAL MARKETING SERVICE, GRAIN DIVISION, USDA; BUREAU OF LABOR STATISTICS)

*PRIOR TO MAY 1972, PRICE IS FOR MILLED NATO AT NEW ORLEANS.

reserve stockpiling and acreage controls or "set-aside" politically attractive.*
These factors include an accumulation of one of the largest U.S. grain surpluses in history after three back-to-back record harvests.

1.2 Previous Studies: Costs and Benefits

The costs of a government grain storage program are substantial: Reutlinger [8] in 1975 estimated that a 20-million-ton program, operated under a storage rule that gives a high degree of protection against likely shortages, would have an annual expected storage cost of $150 million. This program also implies, according to Reutlinger, a net economic loss of $123 million which falls entirely on the shoulders of the producers. The analysis in the Reutlinger paper ignores, however, the benefits of price stabilization. An earlier 1971 study by Tweeten, Kalbfleisch and Lu [2] found that the optimal target for wheat carryover was 400 million bushels (or 10.9 million tons) and that the average cost of storage at this level would be $60 million while net economic loss was $27 million at a $.20 per bushel spread between acquisition and release prices. These figures were based on now outdated wheat supply and demand conditions. A 1974 simulation analysis of grain reserve stock management by Ray, Richardson and Collins [7] used the policy

*New York Times Report, August 30, 1977: "Washington, August 29--In the face of mounting surpluses, President Carter has decided to curb the nation's 1978 wheat crop and give the federal government a bigger role in the grain stockpile business. The cost of the new program was estimated at $4.4 billion.

"At a White House briefing today, Acting Agriculture Secretary John White announced that the president would seek Congressional authorization for a 20 percent cutback in wheat acreage. He signaled administration intentions to get a 10 percent reduction in plantings of feed grains later in the year but said the latter decision was being delayed in case bad weather intervened.

"The program also envisages the placing of 30 million to 35 million tons of food and feed grains in reserve before the beginning of the 1978-79 marketing year. Included in the figure is a proposal to create a special International Emergency Food Reserve of up to six million tons."

rules of Humphrey's Senate Bill 2005 and a $.15 per bushel storage cost for wheat to conclude that price variability would be reduced by 15 percent, and annual storage costs would increase by $30.11 million, while annual deficiency payments would be reduced by $85.33 million. In this study, the mean carryover of wheat would be 483 million bushels or 13 million tons, with government stocks at 77 million bushels. Economic costs and benefits were not presented.

In modeling the welfare effects of a grain reserve policy in the much more complicated case of a multi-national world with international grain trade, while it is still possible to assert that the whole world gains from price stabilization [23], it is no longer clear whether the exporting nation (the United States) gains or loses, and whether the producers or consumers gain or lose. The nonlinearity of the demand curve is central to this issue. Similar doubts exist for importing nations. Hillman, Johnson and Grain [19] stated in 1975 that "(a) demand curves grown steeper at higher prices and shallower at lower prices enhance the consumer stake while diminishing the producer stake in reserves." Just, et al. [23] analyzed the situation of the two-region world, one region exporting the other importing, when the demand curve is highly nonlinear. They conclude that "producers in exporting countries prefer (price) instability, but consumers in importing countries gain from stabilization. Exporting countries are generally worse off and importing countries are better off with stabilization." Since the degree of nonlinearity is shown to affect these conclusions by Just, et al., there is clearly a need for more precise econometric analysis of the grain demand functions, particularly in countries with competitive markets.

In [9], Peter Helmberger and Bob Weaver derived welfare gains and losses to buyers and producers in the context of a "rational expectations" model of price uncertainty. They work with n periods--an initial period of abundance followed by

n-1 periods of normal demand and supply. Government storage programs that stabilize price either completely or partially are studied relative to competitive equilibrium without government storage. Since the international grain trade is not considered, the analysis cannot be considered "realistic." Unlike previous studies, however, the authors do take into account the effect on private storage of the government programs, and their conclusions differ sharply from some previous studies [8,12] in one important respect: the net economic effect of the government storage programs is a loss to society. Their model shows that a massive transfer of benefits* from buyers to grain producers results from a price stabilization program based on government storage of grains.

A major recent study by the International Food Policy Research Institute [24] considers the food security of less-developed countries which, in some years, need to import wheat due to poor harvests. An insurance approach is employed. Sixty-five developed countries were included in the study. Two alternative insurance schemes were evaluated over a five-year period. The rules for release of grains (or funds to pay for food imports) were based on the national food import bill as a percentage of trend value (e.g., 110 percent). A percentage of projected demand (e.g., 95 percent) is established as the target, to be maintained by the international reserve, whenever possible. The study measures the probable cost and the probability of maintaining the target objectives as a function of the size of the grain reserve and the rules for operating it. An excellent feature of the study is that it permits the costs and benefits to be treated stochastically with understanding of the natural year-to-year fluctuation of harvests. This allows trade-off analysis between the cost of the program and the probability of meeting the objectives of the program. Another significant contribution which the study uniquely provides is the disaggregation of the process of providing food security to

*Over $8 billion.

the individual country level in the Third World. The conclusions of this research will undoubtedly be studied closely by food program administrators and policymakers.

1.3 Theory of Price Stabilization Programs Based on Storage of Grains

There are numerous issues, many of them controversial, surrounding the subject of government stockpiling of grains for price stabilization or for humanitarian food aid programs. In attempting to deal with the costs and benefits of a government storage program here, we will focus on a few variables which have been incorporated into analytical models by economists studying grain reserve policy. The first is the domestic demand elasticity and indeed the entire demand function for each specific food and feed grain, wheat for instance. The second is the inherent variability of prices in the market system. The third is the cost of storage. The fourth is the feedback between the market and the storage managers. Export and import demands are also important and so is risk aversion.

The models of government storage program effects discussed here can be classified roughly as follows:

a. Linear and nonlinear demand function

b. Trade exogenous or endogenous to the model

c. Private storage industry considered or not

d. Multiperiod versus single period model

e. Supply and demand uncertainty considered or not.

1.3.1 The Economic Effects of Price Stabilization

Economists are agreed on theoretical grounds that stability of commodity prices conveys benefits to society [10,12,16]. Hayami and Peterson [11] showed the consumers gain, but producers lose twice that amount from a fluctuation in

commodity prices, given a linear demand schedule.* Therefore, society would benefit if that price fluctuation would be reduced at no cost. Subotnick and Houck [12] analyzed the welfare implications of stabilization and showed that price signals for government interventions were superior to quantity (production or consumption) signals. Weaver and Helmberger [9] expressed doubt whether quantity stabilization was feasible at all in the presence of a private storage industry. ECON analyzed the economic benefits of price stabilization brought about by improved crop forecasts using a dynamic welfare optimization [13, 14, 15]. Benton Massell [16], using linear demand and supply schedules

$$S = \alpha p + u$$

$$D = -\beta p + v$$

found a net economic welfare gain or $((\alpha+\beta)/2)\Delta\sigma_p^2$ from a price stabilization measured by a reduction $\Delta\sigma_p^2$ in price variance. He also quantified the producer and consumer benefits (or losses) as follows:

$$\text{Consumer} \qquad \frac{(2\alpha+\beta)\sigma_v^2 - \beta\sigma_u^2}{2(\alpha+\beta)^2}$$

$$\text{Producer} \qquad \frac{(\alpha+2\beta)\sigma_u^2 - \alpha\sigma_v^2}{2(\alpha+\beta)^2}$$

(These results refer to perfect stabilization.) A Rand study [17] concluded, on the basis of a dynamic optimization model, that the long-run standard deviation of wheat prices would fall from \$0.72 per bushel by 15 percent (39 percent) to \$0.61 (\$0.44) per bushel with \$0.15 (\$0.30) subsidy on carryover stocks at an expected cost of \$34 million (\$120 million) annually. The authors point out that a "key

*The actual amounts quoted in their paper were both exaggerated by a factor of two.

parameter is the rate at which government-owned grain stocks would substitute for (and replace) privately owned stocks." This factor is overlooked in most studies, and the consequences for evaluating government stockpiling programs are serious. Helmberger and Weaver [9] account for private storage behavior, and show that with a "rational expectations" approach to price uncertainty there would be substantial gains to producers and losses to buyers from government storage programs designed to stabilize prices. Just, et al. [23] demonstrated analytically the importance of nonlinearity for determining even the correct signs of the welfare effects of government price stabilization in grains.

1.3.2 Government Role in Achieving Price Stabilization

Price stability is apparently socially desirable. Why does private stockholding not accomplish sufficient price stability? Numerous arguments have been advanced. Briefly these include: (1) the discount rates used in evaluating private grain storage investment; (2) risk aversion in the private storage sector; (3) lack of competitiveness of international grain trade and grain storage markets; (4) strict government controls in the European Economic Community, Japan and Russia [22]; (5) producers may prefer price instability [23]; and (6) consumers may prefer price instability [9]. But a very important reason, as pointed out in [19] by Hillman, Johnson and Gray, is that the profit motive cannot be expected to lead to investment in crop failures, which by their nature are somewhat unpredictable and improbable events. Lacking more precise information, private investors will assume an average crop, particularly at an early stage in the crop cycle when no objective data on crop growth exists yet.

In previous ECON studies [13,14,15] we have demonstrated that improved information on crop production worldwide would make a substantial contribution to economic surplus in a free-market world. Given the various distortions introduced

into the market system by governments in their food production and trade policies, it appeared desirable in 1977 to introduce grain reserve stockpiling by the U.S. Government to deal with the huge surpluses, low prices and weak export demand. Regardless of whether this policy is a good one or not, it is important to study the impact of production uncertainty on the management of the government stocks. We observe several points: (1) government stocks must be efficiently managed to achieve their main purpose of price stabilization; (2) good reserve management requires good information; (3) the secondary aim of providing international food aid out of the government grain reserve can only be achieved if a part of the reserve is set aside for this purpose; (4) the food aid can be more effective if good forecasts of foreign crop failures are available in time to the administrator of this program; and (5) the economic costs of the government grain reserve program can be minimized if optimal acquisition and release decisions (timing and amount) are made.

To the extent that a grain reserve goes to provide for food emergencies in importing countries, this grain reserve will fulfill a function otherwise not met by free trade in world commodity markets. This distinction between government stockpiling of grains for purposes of domestic price stabilization (where stocks will be sold on the market at some future time) and on the other hand, food aid for needs otherwise not met (where these stocks simply "disappear" from the market), leads to a fundamentally different assessment of effects and benefits of improved crop information.

1.3.3 The Need for Dynamic Feedback Optimization of Stock Policies

Nearly all the grain stockpiling models which we have reviewed are either static [8,11,23] or simulations over time in which stock levels are set by some

simple rule [2,3,4,7,9,24].* The models also differ on whether the production of grains is treated exogenously or is made responsive to prices, with lags in some cases. From our research at ECON over the past few years, we have discovered that the consequences of ignoring the dynamic nature of the grain economy is very serious; and that the consequences of ignoring the feedback between the market and the grain producers and inventory holders is also serious. Gustafson [20] in his 1958 study for USDA made these same points. He developed a two-period one-world dynamic optimization model with feedback.

Johnson and Sumner [21] calculate optimal grain reserves for developing countries and regions, using a method based on the pioneering Gustafson work. They measure the costs of an "insurance" program for each of a number of developing countries under which they can guarantee themselves adequate food supplies from grain reserves when their own crops fail with a specified probability. Concerning further work along these lines, the authors state: "Some of the most useful generalization of this model might include the incorporation of stochastic demands, nonindependent production probability distributions over time and nonconstant elastic demand curves." Keeler [18] at the RAND Corporation, seems to have developed a dynamic programming approach to optimal distribution, although it is hard to tell from the RAND report. Helmberger and Weaver [9], in the concluding observations of the previously mentioned paper, state: "An important objective of this paper has been to pave the way for more meaningful theoretical and empirical work on the efficiency and distributional consequences of grain storage. The theoretical analysis should be extended through allowing for changing stochastic demand and supply, risk aversion and possible externalities.

*The latter should also be differentiated depending on whether they use a rule of thumb or an estimated behavioral rule.

Precise estimates of demand and supply elasticities are required for many purposes, including the understanding of welfare effects of storage."

Taylor and Talpaz [28] present results of stochastic simulations of a first-order certainty equivalence decision rule for approximately optimal wheat stocks in the United States. Their decision rule is obtained by maximizing a first-order approximation of the discounted sum of expected producers' surplus plus consumers' surplus less storage costs over a long time horizon. The methodology of their study comes closest in essence to the ECON Integrated Model; it does not, however, incorporate the effects of crop forecast error rates.

1.4 Use of ECON Integrated Model for Analysis of Optimal Grain Reserve Management

The ECON Integrated Model is a multiperiod, two-region world and solves the infinite-horizon production and distribution optimization by a combination of dynamic programming and simulation techniques. While this model was developed to measure economic benefits of improved crop information, it is well-suited for studying the optimal management policy for a government grain reserve. Furthermore, because of its capability for handling the effects of crop information, the ECON Integrated Model is valuable for obtaining an analysis of the optimal storage program management policy under supply uncertainty; to our knowledge, no other model has this capability. Figure 1.2 shows a 30-year simulation of U.S. wheat stocks resulting from optimal distribution decisions.

The ECON Integrated Model, with minor modifications, can be used to study the optimal policies for U.S. grain reserve management, given various levels of supply uncertainty (quality of crop production information). For grain price stabilization, the optimal private storage decisions would be supplemented by government storage of wheat for the reserve program or buffer stock. The

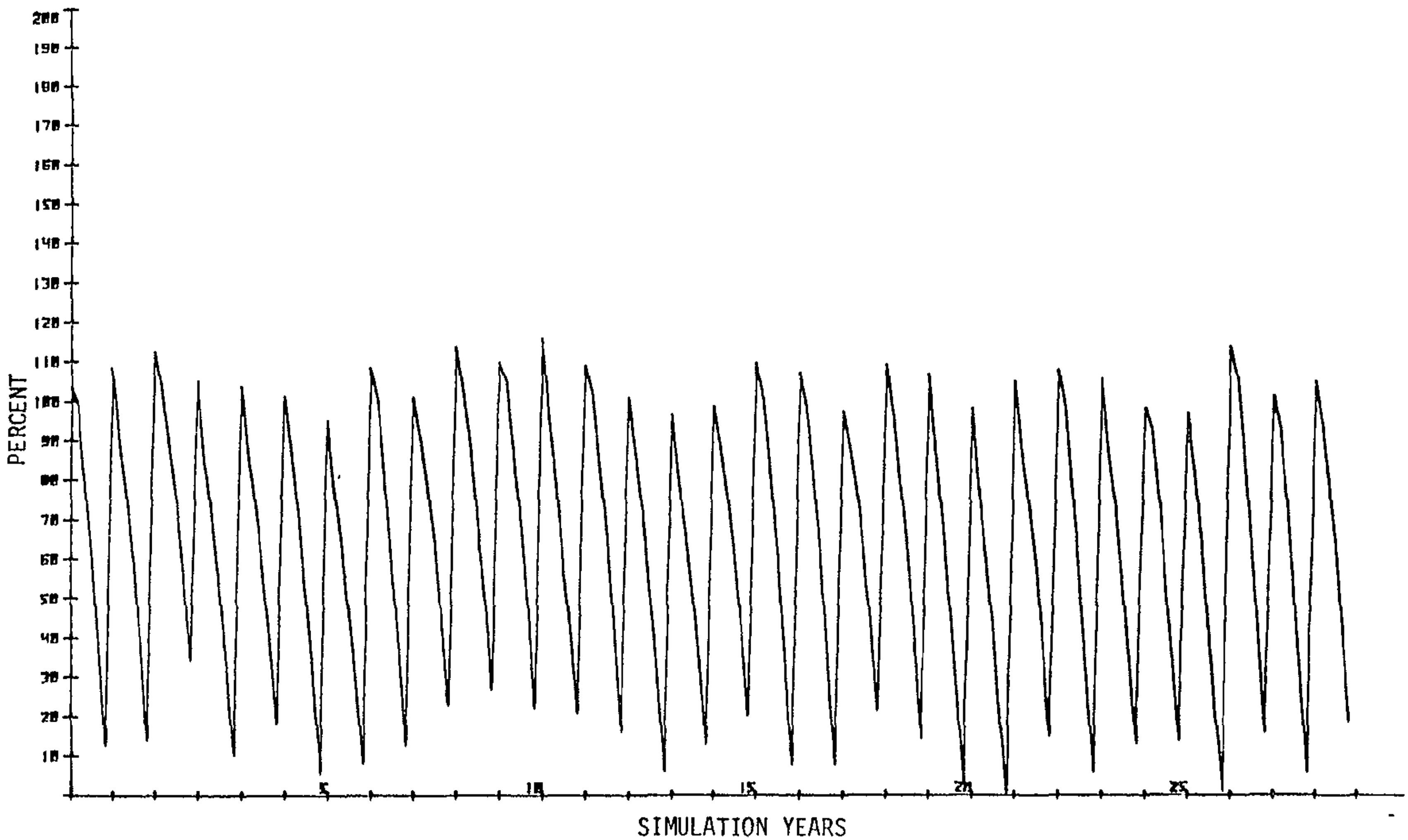


FIGURE 1.2 U.S. STOCKS AS A PERCENT OF PRODUCTION--30-YEAR SIMULATION

government actions would interact through welfare optimization with the private industry production and storage decisions.

Under price stabilization, it is assumed that the reserve will be used to prevent excessively low prices by increased government stockpiling when there are large surpluses, and to keep a ceiling on prices by releasing government stocks to the market in times of grain shortage. The market price is used as a signal for action on the part of the reserve management; government policy is to keep prices within a specified price band. The floor and ceiling prices for this policy are inputs to the modified Integrated Model and they act as constraints on the model's decision making. The net economic welfare, as measured by the model's criteria, is decreased by the policy, but the economic cost of the policy is minimized.

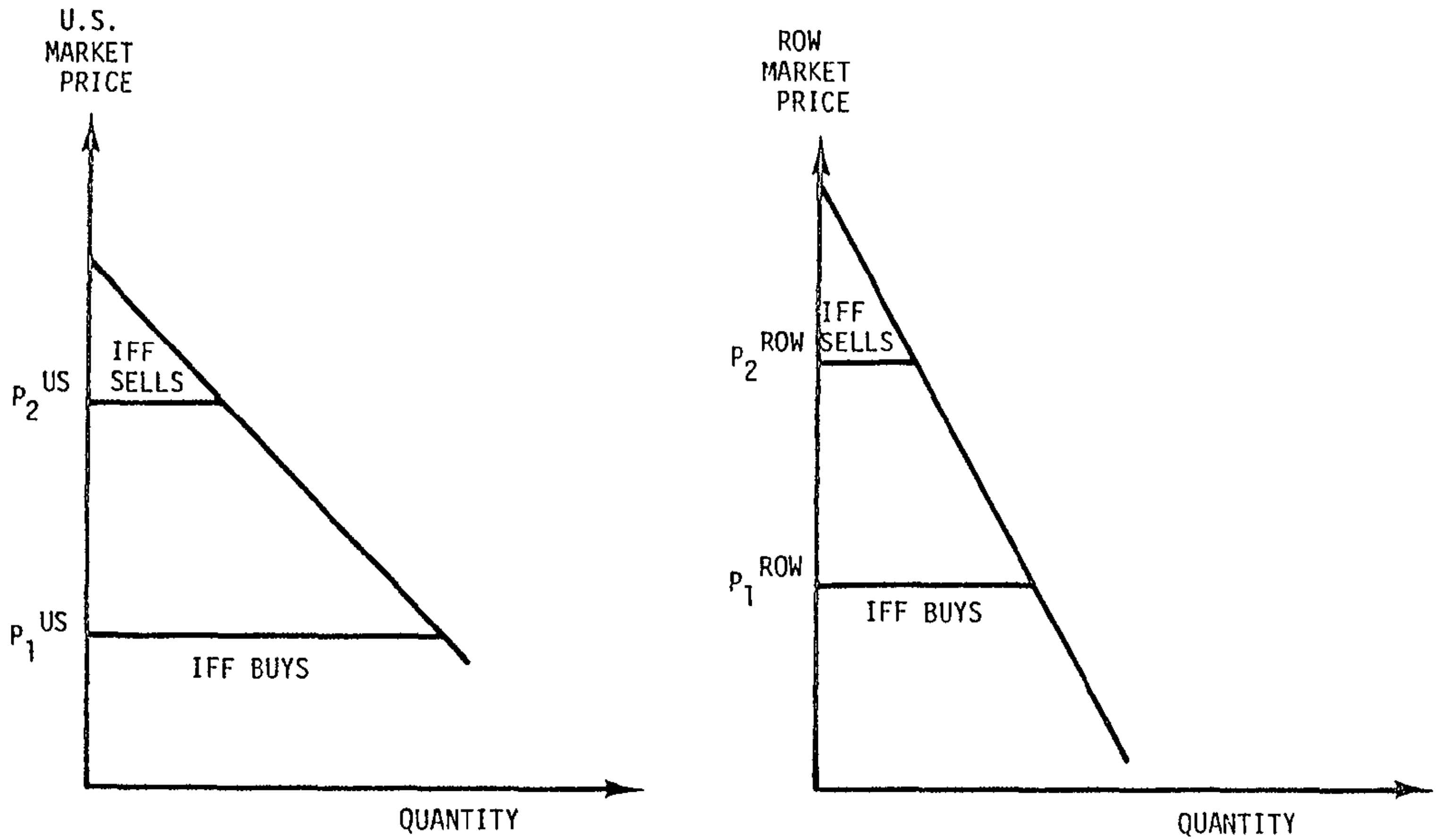
Under food aid programs, the aim of government policy is to provide relief to food-deficient developing countries. Thus, grain is stockpiled to a prespecified level (which may, however, vary as a function market price) and is released under prespecified conditions of food shortage (high prices) in selected countries. After release, the food aid reserve must be rebuilt over a prespecified period of time. The rules for achieving this have a crucial effect on the economic cost of the program, and must be specified in a clear and unambiguous manner before optimization is possible. Once these rules are specified, the model can determine the optimal decisions over time with respect to actual conditions.

1.5 An International Food Fund: Policy Simulation

The ECON Integrated Model has been used in this study to simulate the operation of an International Food Fund (IFF). The sizable wheat stocks which are acquired by the fund at the beginning are used to supply wheat when shortages develop as signaled by high market prices. During periods of surplus, as indicated by low market prices, the IFF replenishes its stocks, buying only sufficient wheat

to bring the market price up to a support level. Recalling that the ECON Integrated Model has two regions, the United States and Rest-of-World (ROW), there are two distinct wheat markets and hence two distinct price bands to consider. The IFF in our simulation will buy and sell wheat in either market whenever the prices trigger such actions by reaching either floor or ceiling levels (see Figure 1.3). The wheat in the IFF stocks may be released to either region as needed regardless of its origin. Transportation costs are always paid by importers as in the Integrated Model itself. In effect, we have assumed that the IFF wheat stocks are stored in the country which produced the wheat and are only transported when needed for consumption. Most of the surpluses, of course, occur in the United States, while most of the shortages occur in the ROW. We have not attempted to deal with the problem of IFF financing. The initial purchase of, for example, 27.11 MMT of U.S. wheat at a price of $140/MT requires a capitalization of $3.795 billion. At a 15 percent annual carrying charge, such a fund has 25-year total costs, the present worth of which under current information are $1.5 billion, allowing for continuous purchases at $140 ($155) in the U.S. (ROW) market and sales at $220 ($250) and eventual disposal of the remaining stocks at the prevailing market prices. The present worth of ROW consumer benefits under current information over the 25 years is $1.4 billion. The 15 percent carrying charge includes storage costs of $0.625 per metric ton per month or 5 percent per annum at average prices and interest charges of 10 percent per annum. Detailed results are presented in Chapter 5 for various sizes of initial stocks and various price bands.

In presenting this parametric policy simulation we are using the Integrated Model in two ways; (1) to optimize the "free market" model for each information



UNITED STATES DEMAND RELATIONSHIP WITH IFF INVOLVEMENT

REST-OF-WORLD DEMAND RELATIONSHIP WITH IFF INVOLVEMENT

FIGURE 1.3 PRICE BANDS FOR UNITED STATES AND ROW WITH INTERNATIONAL FOOD FUND (IFF)

system (with and without LANDSAT); output of this optimization is a set of state-variable statistics and coefficients of the steady state-value function for each information system; (2) to simulate the IFF operations, costs and benefits over many years, using the optimized coefficients from (1). Note, however, that the bimonthly decisions on planting, consumption, trade and private storage are locally optimized with respect to the fixed economic value function and state variable statistics. There is accordingly "local feedback" from the IFF operations to the market. For a treatment of the more ambitious undertaking of solving the Integrated Model with IFF policy as constraints on the global optimization, see the discussion in Chapter 2. This problem has not been fully implemented at the time of writing.

2. FORMULATION OF QUADRATIC PROGRAMMING PROBLEM

In order to implement the additional constraints of a price ceiling and price floor within the existing framework of the integrated model, several alternatives were considered. All of the alternatives explicitly incorporated government intervention in the form of buying and selling wheat to aid in the stabilization of wheat prices. Two of the options seemed superior and were given further consideration:

1. A fixed price floor and fixed price ceiling
2. A penalty cost for violating the price floor or price ceiling.

The first case occurs when the government intervenes by selling from government stocks when the price is high and buying in the marketplace when the price is low. The constraint allows government action only when the price achieves the ceiling (floor) and the amount of the sale (purchase) would be strictly determined as the amount required to exactly maintain the price ceiling (floor).

This approach shows two major weaknesses: one theoretic and one algorithmic. The theoretic weakness is that if the price achieves the ceiling and the government is forced to sell, sufficient government stocks might not exist to allow for the stabilization of the price.* In this case, there is no feasible solution to the problem without relaxing the price constraint or adjusting government stocks artificially. In other words, some alternative action plan would be necessary since the problem as defined could not be solved.

The algorithmic problem with the first approach is one of practicality. In the original statement of the problem, all of the constraints on the system were linear,

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*A similar problem exists at the price floor.

thus allowing for the use of straightforward quadratic programming techniques. The proposed first approach not only would increase the dimensionality of the problem, but also would change the constraint set from linear to nonlinear, a significant change in terms of the applicable solution algorithms and execution times. Since the execution time of the model is a major consideration, and since another viable alternative existed, the first approach was rejected.

The proposed method for implementing price stabilization constraints by government intervention on the integrated model is to impose a penalty on any violation of the price bounds. A severe penalty cost will imply that the bounds will be maintained whenever feasible and a zero penalty will imply that the prices will fluctuate as in the current unconstrained manner. By using the penalty function approach, the two weaknesses of the first approach are avoided. For example, if sufficient government stocks are not available for sale in case of high prices, then the price ceiling would be exceeded and a penalty cost would be charged. The selection of an appropriate penalty is a subject of significant interest but cannot be fully discussed here. In addition, the penalty function can be defined as linear and the constraint set of the expanded problem would remain linear, thus avoiding the algorithmic weakness of the nonlinearity of the first approach.

The price constraints are to be incorporated within the existing model which is fully described in Reference 15, particularly Chapter 4. In order to maintain continuity in this report and consistency with Reference 15, a shortened description of the model is presented below in which both the terminology and the variables of the previous report will be retained. Following the discussion of the existing model, the formulation of the modifications that would be necessary to incorporate government intervention as price control will be described.

2.1 Existing Model Definition

In the model, the year is divided into six periods and export decisions are made simultaneously with consumption versus storage decisions at the beginning of each period. Two regions are considered, called the exporting unit (the United States) and the importing unit (ROW). In each region, planting decisions are made at specific times of the year, depending on the crop under study. For wheat, spring and winter sowing are distinguished, and the Southern Hemisphere sowing occurs half a year out of phase with Northern Hemisphere winter sowing.

State Variables

At time 1, the beginning of the first period, there are two state variables. The first, x_1, refers to the mean value at time 1 of stocks in the exporting unit, including the newly available production (still uncertain) and the carryover from the previous crop year (known). The second state variable refers to the mean value at time 1 of stocks in the importing unit. From time 1 until the start of the period after the first planting, the same two state variables are used to track the state of the system. At each time during this interval, x_1 refers to the mean value of remaining supply in the exporting unit, after accumulated consumption and accumulated exports; x_3 refers to the mean value of remaining supply in the importing unit, including imports and after accumulated consumption. When the first planting occurs in either unit, an additional state variable is created to represent the mean value of the production expected to result from the planting in the following crop year. Thus, there may be three or four state variables in the middle periods of the crop year, and there will be four state variables by the end of the crop year. When planting has occurred in the exporting unit, the new state variable is denoted x_2. When planting has occurred in the importing unit, the new state variable is denoted x_4.

Decision Variables

The vector of decision variables, like the state vector, has fluctuating dimension. There are always at least three decision variables. They are: y_1, consumption in the exporting unit; y_2, exports; and y_4, consumption in the importing unit. In the planting periods for the exporting unit, there is also the intended production, y_3, and in the planting periods for the importing unit, there is the intended production, y_5.

State Transformation

The state vector undergoes a change from one time to the next as a result of decisions and new information on existing or potential (planted) supply. The vector Φ is used to represent new information. Its elements are random variables of zero mean. Using subscripts to indicate time, we can write the state transformation in vector form as

$$X_{t+1} = M_t X_t + N_t Y_t + \Phi_t.$$

The structure of M_t, N_t, and Φ_t, depend on the planting schedule for the particular crop. In the case of wheat, we will model planting as occurring in periods 2 and 5 in the exporting unit (United States), and in periods 2, 5 and 6 in the importing unit (ROW). For this case, there are six periods, and the year begins June 1. The state transformation matrices are as given in Table 2.1.

Value Functions

We are fundamentally concerned with a cumulative value function, the maximization of which is assumed to govern all decision making. The gross value associated with one period's consumption y_1 in the exporting unit is approximated by the polynomial

TABLE 2.1 STATE TRANSFORMATION MATRICES FOR WHEAT PRODUCTION AND DISTRIBUTION

PERIOD	M	N
1	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 & -0 \\ 0 & 1 & -1 \end{pmatrix}$
2	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
3	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
4	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
5	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
6	$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 1 \end{pmatrix}$

$$\alpha_1 y_1^2 + \beta_1 y_1,$$

and the gross value associated with one period's consumption y_4 in the importing unit is approximated by

$$\alpha_2 y_4^2 + \beta_2 y_4.$$

These are consistent with linear demand functions

$$\text{Price} = 2\alpha_1 y_1 + \beta_1$$

and

$$\text{Price} = 2\alpha_2 y_4 + \beta_2$$

in the exporting unit and the importing unit respectively. The transportation costs and production costs are also approximated by second degree polynomials as follows:

$$\text{Transportation Cost} = \tau y_2^2 + \omega y_2,$$

$$\text{Production Cost} = \gamma_{k1} y_3^2 + \delta_{k1} y_3 + \gamma_{k2} y_5^2 + \delta_{k2} y_5.$$

Here, the subscript k distinguishes the various periods within the year. The net incremental value function for period k now can be written

$$F(y_1, y_2, y_3, y_4, y_5) = \alpha_1 y_1^2 + \beta_1 y_1 + \alpha_2 y_4^2 + \beta_2 y_4 - \tau y_2^2 - \omega y_2$$
$$- \gamma_{k1} y_3^2 - \delta_{k1} y_3 - \gamma_{k2} y_5^2 - \delta_{k2} y_5.$$

Algebraically, we denote the six incremental value functions described in Table 2.2 $F_{1k}, F_{2k}, \ldots, F_{6k}$, where k is the period of the year. These functions can be expressed in terms of coefficient matrices as follows:

$$F_{ik}(Y) = Y'A_{ik}Y + Y'B_{ik} \tag{2.1}$$

where A_{ik} are 5 x 5 matrices and B_{ik} are vectors of five components. These coefficients are collected in Table 2.2.

The fundamental cumulative value function at time t, which is the discounted sum of F_{1k}'s, $k \geq t$, will be denoted V_{1t}. The auxiliary value functions, associated with $F_{2k}, \ldots, F_{6k}$, will be denoted $V_{2t}, \ldots, V_{6t}$. Each is approximated by a second degree polynomial in X_t, as follows:

$$V_{it}(X_t) = X'_t Q_{it} X_t + X'_t L_{it} + K_{it}, \tag{2.2}$$

where each Q_{it} is a symmetric matrix, L_{it} is a vector, and K_{it} is a scalar. Our basic computational task is to find Q_{it} and L_{it}, since this will enable us to determine the dependence of $V_{it}(X_t)$ on the stochastic terms Φ_t.

Dynamic Programming

The optimality principle for the system we are modeling can be written

$$V_{1t}(X_t) = \max_Y \left\{ F_{1t}(Y) + \overline{\rho V_{1(t+1)}(X_{t+1})} \right\} \tag{2.3}$$

TABLE 2.2 INCREMENTAL VALUE FUNCTION COEFFICIENTS

NO.	NAME	A	B
1	TOTAL	$\begin{pmatrix} \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & -\tau & 0 & 0 & 0 \\ 0 & 0 & -\gamma_{k1} & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & 0 \\ 0 & 0 & 0 & 0 & -\gamma_{k2} \end{pmatrix}$	$\begin{pmatrix} \beta_1 \\ -\omega \\ -\delta_{k1} \\ \beta_2 \\ \delta_{k2} \end{pmatrix}$
2	EXPORTER NET WELFARE	$\begin{pmatrix} \alpha_1 & \alpha_1 & 0 & 0 & 0 \\ \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\gamma_{k1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} \beta_2 \\ \beta_2 \\ -\delta_{k1} \\ 0 \\ 0 \end{pmatrix}$
3	EXPORTER CONSUMERS' GAIN	$\begin{pmatrix} -\alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$
4	EXPORTER PRODUCERS' GAIN	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_{k1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$
5	IMPORTER CONSUMERS' GAIN	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$
6	IMPORTER PRODUCERS' GAIN	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_{k2} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

where Y is subject to the constraints

$$Y \geq 0, \qquad y_1 + y_2 \leq x_1, \qquad y_4 \leq x_2.$$

As before, the bar indicates the mean value with respect to the random variable Φ_t. Using equations 2.1 and 2.2, the maximand can be written

$$Y'E_1Y + Y'F_1 + G_1$$

where

$$E_1 = A_{1t} + \rho N'_t Q_{1(t+1)} N_t, \tag{2.4}$$

$$F_1 = B_{1t} + 2\rho Q_{1(t+1)} M_t X N_t + \rho L_{1(t+1)} N_t,$$

$$G_1 = \rho \left[X'M'_t Q_{1(t+1)} M_t X + L_{1(t+1)} M_t X + K_{1(t+1)} + \overline{\Phi'_t Q_{1(t+1)} \Phi_t} \right].$$

The evaluation of equation 2.3 for a given value of X is thus a quadratic programming problem with five variables and two constraints. It can be solved easily, provided the values of $Q_{1(t+1)}$, $L_{1(t+1)}$ and $K_{1(t+1)}$ are known. If it happens that the constraints on Y in this maximization are not encountered, then the maximizer Y* is given by

$$Y^* = -1/2 E_1^{-1} F_1,$$

so that V_{1t} is a quadratic function of X. If this were always the case, we could expand V_{1t} explicitly in X and read off its coefficients Q_{1t}, L_{1k} and K_{1t}. However, the constraints may be encountered, so V_{1t} is not quadratic in X. To approximate it by a quadratic form, we select a grid of points $X_1, \ldots, X_n$ in the state space at time t. We evaluate V_{1t} at each of these points (by quadratic programming), and then determine the coefficients Q_{1t}, L_{1t} and K_{1t} of the quadratic polynomial giving the least squares fit to V_{1t} at the selected points. This two-step procedure--quadratic programming followed by least squares approximation--provides value function coefficients Q_{1t}, L_{1t} and K_{1t}, assuming $Q_{1(t+1)}$, $L_{1(t+1)}$ and $K_{1(t+1)}$ are known. At the same time, the procedure is used to obtain the auxiliary value function coefficients Q_{it}, L_{it} and K_{it} for i = 2, ..., 6, since $V_{it}(X)$ are given by

$$V_{it}(X) = Y^{*\prime}E_i Y^* + Y^{*\prime}F_i + G_i$$

where Y* is the maximizer in equation 2.3 and

$$E_i = A_{it} + \rho\, N_t' Q_{i(t+1)} N_t,$$

$$F_i = B_{it} + 2\rho Q_{i(t+1)} M_t X N_t + \rho L_{i(t+1)} N_t,$$

$$G_i = \rho \left[X' M_t' Q_{i(t+1)} M_t X + L_{i(t+1)} M_t X + K_{i(t+1)} + \overline{\Phi_t' Q_{i(t+1)} \Phi_t} \right].$$

Thus, the five auxiliary value functions are approximated by least squares on the same grid as is used for V_{1t}.

Starting with terminal value assumptions on V_{i1}, $i = 1, \ldots, 6$, corresponding to some year far in the future, we can repeat the backward induction steps described above to obtain first V_{im}, $i = 1, \ldots,$ then $V_{i(m-1)}$, $i = 1, \ldots, 6$, etc. After m steps, we obtain a new set V_{im}, $i = 1, \ldots, 6$, this time corresponding to one year earlier. Continuing the cycle through the m periods each year until we get back to the present, we finally obtain the desired functions. Because of the use of discounting, it makes no difference what terminal value assumptions are made, provided we begin the backward induction far enough in the future.

Another viewpoint on the same calculation is the following. Because we are building a steady state model, the value functions should be identical at times one year apart. Thus, for any fixed X, $V_{it}(X) = V_{i(t+m)}(X)$. If Γ stands for m steps of backward induction as described above, then we must have

$$\Gamma V_{it} = V_{it},\ i = 1, \ldots, 6;\ t = 1, \ldots, m.$$

Starting with any approximations V_{it}^{1}, we can produce a sequence of approximations $V_{it}^{1}, V_{it}^{2}, V_{it}^{3}, \ldots$, by repeating Γ. Thus,

$$V_{it}^{n+1} = \Gamma V_{it}^{n}.$$

When successive approximations are close enough to equal, they can be taken as the solution of

$$\Gamma V_{it} = V_{it}.$$

This convergence does occur in the model, because of the presence of the discount factor ρ in the optimality principle (2.3).

Grid for Value Function Approximation

As mentioned above, a grid of points $X_1, \ldots, X_n$ is selected in the state space for each time t, t = 1, ..., m. Initially, these grid points are selected by judgment, so that the points cover the expected range of variation of the state vector. After solution of the optimality principle (equation 2.4), we can explicitly evaluate the state transformation, and thus track the development of the state vector through many years. Using Monte Carlo simulation, we find the probability distribution of the state vector for each time t. Then we adjust the grid points to conform to this distribution, and repeat the procedure. This sequence--solution of optimality principle followed by simulation--is continued until convergence is attained.

At a given time t, the grid represents a discrete equiprobable distribution. If d is the dimension of the state space at time t, and n is the number of values of each coordinate represented in the grid, there are n^d points in the grid. In each coordinate, the values are equally spaced. Such a grid is completely determined by the mean and standard deviation of each coordinate. Thus, only these statistics are collected from the simulations and convergence is considered to be achieved when the means and standard deviation of each coordinate at each time of year have stabilized.

2.2 Modifications to the Model

The constraints which are to be added to the model take the following form:

Let

PF = price floor

PC = price ceiling.

Then the desired constraints on domestic wheat prices are

$$PF \leq \text{Price} \leq PC.$$

Since the price is defined as

$$\text{Price} = 2\alpha_1 y_1 + \beta_1$$

then, in terms of the model, the constraints become

$$2\alpha_1 y_1 + \beta_1 \leq PC \tag{2.5}$$

$$2\alpha_1 y_1 + \beta_1 \geq PF. \tag{2.6}$$

If we define two "slack" decision variables S_1, S_2 as

$0 \leq S_1$ = amount below the price ceiling

$0 \leq S_2$ = amount above the price floor

then the inequalities (2.5), (2.6) become

$$2\alpha_1 y_1 + \beta_1 + S_1 = PC \tag{2.7}$$

$$2\alpha_1 y_1 + \beta_1 - S_2 = PF. \tag{2.8}$$

Since the intent of the formulation is to allow the constraints to be violated at some linear penalty cost, then we introduce two additional decision variables

$0 \leq V_1$ = amount above the price ceiling

$0 \leq V_2$ = amount below the price floor.

Given V_1 and V_2, equations (2.7) and (2.8) can be rewritten as

$$2\alpha_1 y_1 + \beta_1 + S_1 - V_1 = PC \tag{2.9}$$

$$2\alpha_1 y_1 + \beta_1 - S_2 + V_2 = PF. \tag{2.10}$$

If the unit penalties for violating the respective constraints are defined as

Π_1 = unit penalty for violating price ceiling

Π_2 = unit penalty for violating price floor

then the total associated penalty cost would be

$$\text{Penalty Cost} = \Pi_1 V_1 + \Pi_2 V_2 . \tag{2.11}$$

Since equations (2.9), (2.10) and (2.11) are all defined in terms of prices, it is necessary to translate the prices into quantities of wheat in order to incorporate government buying and selling as a price stabilization control. After some minor algebraic manipulation, equations (2.9) and (2.10) become

$$y_1 + \frac{1}{2\alpha_1} S_1 - \frac{1}{2\alpha_1} V_1 = \frac{(PC - \beta_1)}{2\alpha_1} \tag{2.12}$$

$$y_1 - \frac{1}{2\alpha_1} S_2 + \frac{1}{2\alpha_1} V_2 = \frac{(PF - \beta_1)}{2\alpha_1} . \tag{2.13}$$

Now $\frac{1}{2\alpha_1} V_1$ and $\frac{1}{2\alpha_1} V_2$ are the quantities below the quantity floor and above the quantity ceiling and consist of government transactions plus violations in excess of government transactions. Let

$0 \leq y_6$ = government sales from stocks

$0 \leq y_7$ = government purchases into stocks.

Now

$$y_6 \leq \frac{1}{2\alpha_1} V_1$$

$$y_7 \leq \frac{1}{2\alpha_1} V_2$$

or

$$y_6 - \frac{1}{2\alpha_1} V_1 + S_3 = 0 \tag{2.14}$$

$$Y_7 - \frac{1}{2\alpha_1} V_2 + S_4 = 0 \tag{2.15}$$

where S_3, S_4 are nonnegative slack variables representing the constraint violations in excess of the government transactions with respective costs Π_3 and Π_4. Since the government sells at the price ceiling and buys at the price floor, the associated cost of equations (2.14) and (2.15) is

$$-PC \cdot y_6 + PF \cdot y_7 + \Pi_1 V_1 + \Pi_2 V_2 + \Pi_3 S_3 + \Pi_4 S_4.$$

For simplicity we can assume $\Pi_1 = \Pi_2 = \Pi_3 = \Pi_4 = \Pi$ where Π is a very high penalty cost.

Notice that since

$$y_6 + S_3 = \frac{1}{2\alpha_1} V_1 \qquad\qquad S_3 = \frac{1}{2\alpha_1} V_1 - Y_6$$

or

$$y_7 + S_4 = \frac{1}{2\alpha_1} V_2 \qquad\qquad S_4 = \frac{1}{2\alpha_1} V_2 - y_7$$

the penalty costs are double counted as

$$-PC \cdot y_6 + PF \cdot y_7 + \Pi V_1 + \Pi V_2 + \Pi\left(\frac{1}{2\alpha_1} V_1 - y_6\right) + \Pi\left(\frac{1}{2\alpha_1} V_2 - y_7\right)$$

or

$$y_6(\Pi - PC) + y_7(\Pi + PF) + V_1 \Pi\left(1 + \frac{1}{2\alpha_1}\right) + V_2 \Pi\left(1 + \frac{1}{2\alpha_1}\right)$$

or

$$y_6(\Pi - PC) + y_7(\Pi + PF) + (V_1 + V_2)\ \frac{\Pi}{2\alpha_1}\ (1 + 2\alpha_1).$$

The value of the objective function must be adjusted after the optimization to adjust for the artificial penalty costs

$$\Pi\left[y_6 + y_7 + (V_1 + V_2)\left(1 + \frac{1}{2\alpha_1}\right)\right].$$

In order to avoid any inconsistencies in the problem definition, we must restrict government sales and purchases to be within some feasible region. That is to say, government sales must be less than or equal to government stocks and government purchases must be less than or equal to existing stocks (which includes newly available production and the carryover from the previous year) minus consumption minus exports. Let

$0 \leq X_5$ = government stocks.

Then

$$y_6 \leq X_5$$

$$y_7 \leq X_1 - Y_1 - Y_2$$

or

$$y_6 \leq X_5$$

$$Y_1 + Y_2 + Y_7 \leq X_1$$

by adding slack variables, we get the equations

$$y_6 + S_5 = X_5 \tag{2.16}$$

$$y_1 + y_2 + y_7 + S_6 = X_1. \tag{2.17}$$

Now combining the entire constraint set we have

$$y_1 + y_2 + S_7 = X_1 \tag{2.18}$$

$$y_4 + S_8 = X_2 \tag{2.19}$$

$$y_1 + \frac{1}{2\alpha_1} S_1 - \frac{1}{2\alpha_1} V_1 = \frac{(PC-\beta_1)}{2\alpha_1} \tag{2.20}$$

$$y_1 - \frac{1}{2\alpha_1} S_2 + \frac{1}{2\alpha_1} V_2 = \frac{(PF-\beta_1)}{2\alpha_1} \tag{2.21}$$

$$y_6 - \frac{1}{2\alpha_1} V_1 + S_3 = 0 \tag{2.22}$$

$$y_7 - \frac{1}{2\alpha_I} V_2 + S_4 = 0 \tag{2.23}$$

$$y_6 + S_5 = X_5 \tag{2.24}$$

$$y_1 + y_2 + y_7 + S_6 = X_1 \tag{2.25}$$

and the state transformation for X_5 would be

$$X_5^{t+1} = X_5^{t} + y_6^{t+1} - y_7^{t+1}. \tag{2.26}$$

The incremental value function would now be

$$\alpha_1 y_1^2 + \beta_1 y_1 + \alpha_2 y_4^2 + \beta_2 y_4 - \tau y_2^2 - w\, y_2$$

$$- \gamma_{k1} y_3^2 - \delta_{k1} y_3 - \gamma_{k2} y_5^2 - \delta_{k2} y_5$$

$$- PC \cdot y_6 + PF \cdot y_2 + \Pi(V_1 + V_2 + S_3 + S_4).$$

In terms of the total model modification, the number of decision variablès goes from 7 to 17 and the number of constraints goes from 2 to 8, a considerable enlargement of the problem.

2.3 Potential Problems With Formulation

The primary problem with the model as presented in Section 2.1 is that the computer time necessary for convergence is large, thereby making numerous sensitivity runs impractical. In fact, the model which was originally programmed using the language APL was translated into FORTRAN in order to reduce the execution time and associated costs. Although the costs of the FORTRAN version are significantly less expensive than the costs of the APL version, the execution time of the model remains the major system constraint.

As discussed in Section 2.2, three major changes would be made to the model: (1) the state space would increase from 4 to 5, (2) the constraints would increase

from 2 to 7, and (3) the decision variables would increase from 7 to 17. Currently the state space is approximated by three values in each dimension so that the existing model has

$$3^4 = 81$$

grid points. If the same number of values in each dimension is maintained in the proposed modification, the number of grid points will increase threefold to

$$3^5 = 243.$$

This would imply that the number of evaluations would be increased threefold in the dynamic programming and the computer time would increase similarly.

The current algorithm used to solve the quadratic programming problem requires a tableau of the size

$$\text{Number of rows} = N_{DV} + N_C$$

$$\text{Number of columns} = 2 \cdot N_{DV} + N_C$$

where

N_{DV} = number of decision variables

N_C = number of constraints.

Thus, the size of the tableau in the current model is 9 x 16, whereas the size of the tableau using the proposed modification is 24 x 41, an increase of more than sixfold the number of entries in the tableau. Thus, the quadratic programming algorithm which is in the center of the simulation will be enlarged significantly, thereby increasing the necessary computer time.

It is clear from the above discussion that the implementation of price bounds by government intervention is clearly a feasible project. The major drawback is the expected increase in the computer resources that would be necessary to fully implement the model changes. With further optimization of the FORTRAN code, it is expected that the full implementation will be a reasonable task. Currently the

work is beyond the scope of the existing project and several more simplified alternative implementations were considered. In the following sections, two of the alternatives are discussed.

3. PRICE FLOOR IMPLEMENTATION

The desired price constraints are of the form

price floor $\leq$ price $\leq$ price ceiling

or

$$PF \leq 2\alpha_1 y_1 + \beta_1 \leq PC.$$

By algebraic manipulation (recall $\alpha_1 < 0$) we get

$$QC = \frac{PF-\beta_1}{2\alpha_1} \geq y_1 \geq \frac{PC-\beta_1}{2\alpha_1} = QF$$

or we translate the price constraints into quantity constraints where

QC = quantity ceiling

QF = quantity floor.

Avoiding the question of government intervention, we can assume that the price constraints can be directly translated into quantity constraints as follows

$$QF \leq y_1 \leq QC. \tag{3.1}$$

The major problem with directly implementing this constraint (3.1) is whether it is considered with the existing model constraints. From Section 2.1 we know that the only other constraint involving y_1 is

$$y_1 + y_2 \leq X_1. \tag{3.2}$$

Graphically, we can see that there are two cases of interest for the feasible region on y_1, the case when $X_1 \geq QF$ and the case where $X_1 < QF$. Figure 3.1 shows the case when $X_1 \geq QF$. Notice that the feasible region of y_1 is the shaded triangle and the statement of the problem is consistent. The problem arises when $X_1 < QF$ and Figure 3.2 illustrates the constraints. Notice that there is no region which

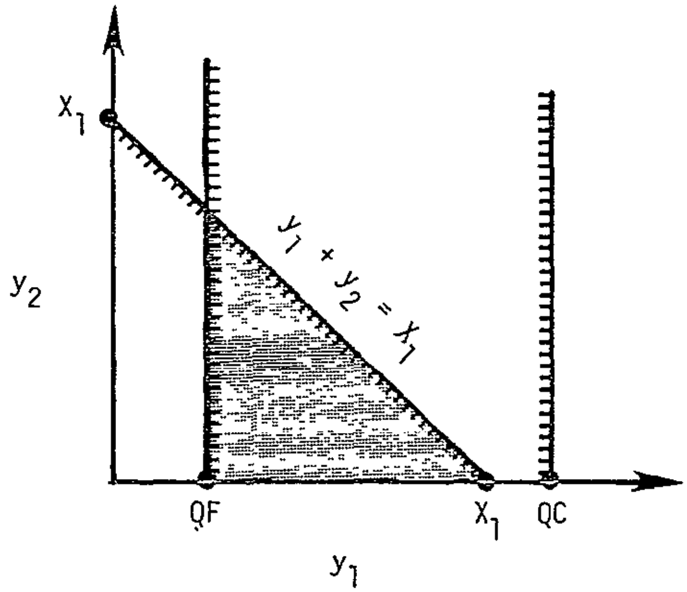

FIGURE 3.1 CASE I: $X_1 \geq QF$

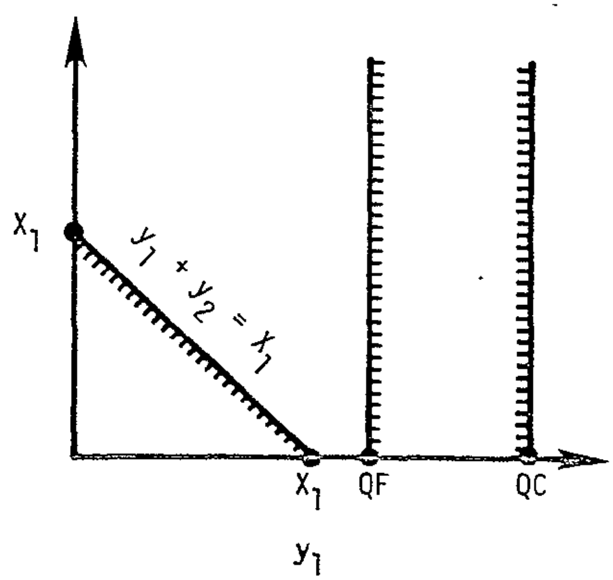

FIGURE 3.2 CASE II: $X_1 < QF$

satisfies both (3.1) and (3.2). Thus, there is an inconsistency in the problem and a theoretic infeasibility in the problem definition.

In both of the above cases the quantity ceiling did not adversely affect the outcome of the feasible region. In fact, there is no value of the quantity ceiling other than zero which would cause an infeasibility. Thus, the implementation of a quantity ceiling, which implies a price floor, does not present any conceptual problems. The question arises, however, of whether the quantity floor (price ceiling) would ever cause the infeasibility as shown in Figure 3.2. In virtually every test of the model, and given a reasonable price ceiling, it was found that, in practice, the problem becomes infeasible a significant number of times to cause concern.

Since the price floor is the price bound of the most interest and since it relates to the 90 percent parity demands of farmers, a version of the model was created which implemented only the price floor constraint. In effect, this constraint is implemented in the model as the quantity constraint

$$y_I \leq \frac{PF-\beta_I}{2\alpha_I}$$

thereby increasing the size of the quadratic programming tableau from 9 x 16 to 10 x 17 and not changing the number of decision variables. This version is available but sufficient runs have not been made to determine the effect of the constraint on the value function.

4. SIMULATION IMPLEMENTATION

The most straightforward implementation of the model with government intervention is that of including government actions only in the final simulation, after the value functions have been documented. The simulation is run in the usual manner and the government transactions are recorded parallel to the model. The value function coefficients are those obtained in the convergence process of the model without the grain reserve program. Thus, the quadratic, linear and constant terms of the objective function in the optimization are those obtained in a free market economy and do not incorporate the changes that would be a result of government intervention. The primary inputs of the simulation runs are the value function coefficients and the initial state values of X_1, the mean value of remaining supply in the exporting unit, and X_3, the mean value of remaining supply in the importing unit.

The form of the constraints in this implementation is

price floor $\leq$ market price $\leq$ price ceiling.

We will simplify the presentation in this section by referring only to the U.S. case; similar equations and inequalities must be added to the ROW constraint set. Thus,

$$PF \leq 2\alpha_1 y_1 + \beta_1 \leq PC \tag{4.1}$$

where $2\alpha_1$ and β_1 are the slope and intercept of the domestic demand curve and y_1 is the decision variable representing the level of domestic consumption. Figure 4.1 gives a graphical representation of the linear demand curve and the price constraints. Notice that the price ceiling implies a quantity floor and that a price floor implies a quantity ceiling. Thus, we wish to constrain consumption as

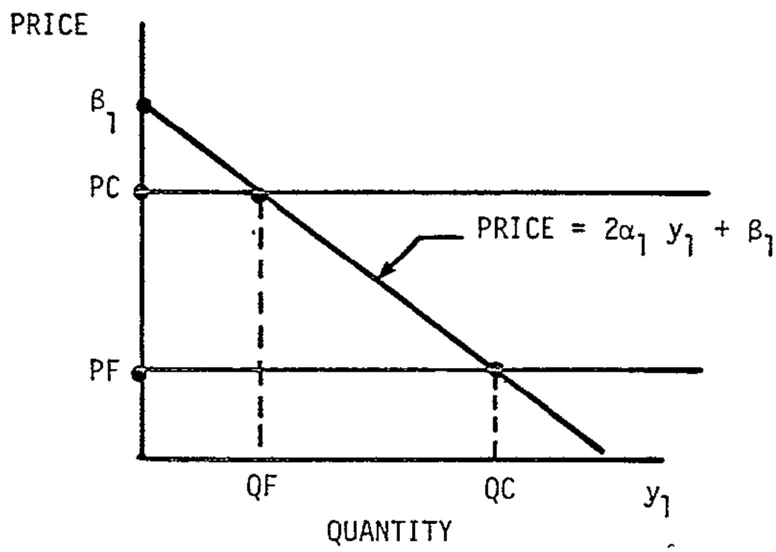

FIGURE 4.1 PRICE CONSTRAINTS ON DEMAND CURVE

$$QF \leq y_1 \leq QC \tag{4.2}$$

which is strictly equivalent to the price constraint (4.1).

The major simplifying assumption made in the simulation implementation is that government actions in the marketplace act exactly like consumer actions in terms of both the value functions and on the price level. That is to say that government sales and purchases are strictly additive to domestic consumption when viewed in relation to the demand curve.

More precisely we assume that

$$\text{domestic price} = 2\alpha_1 (y_1 + y_6 - y_7) + \beta_1 \tag{4.3}$$

where

$0 \leq y_6$ = government sales from stocks

$0 \leq y_7$ = government purchases into stocks.

Since we want

$$PF \leq 2\alpha_1 (y_1 + y_6 - y_7) + \beta_1 \leq PC \tag{4.4}$$

and since we define

$$y_6 \cdot y_7 = 0$$

we will have the government intervene <u>only</u> when

$$2\alpha_1 y_1^* + \beta_1 > PC \tag{4.5}$$

or

$$2\alpha_1 y_1^* + \beta_1 < PF \tag{4.6}$$

where y_1^* is the optimal value of y_1, as determined in the model.

The level of government action required to satisfy the price constraints whenever either (4.5) or (4.6) occurs is now strictly determined. Consider the case of the price exceeding the price ceiling by ΔP. Then

$$2\alpha_1 y_1^* + \beta_1 = PC + \Delta P. \tag{4.7}$$

Using (4.3) we know that

$$2\alpha_1(y_1^* + y_6 - y_7) + \beta_1 = PC. \tag{4.8}$$

Subtracting (4.8) from (4.7) we get

$$2\alpha_1(y_7 - y_6) = \Delta P$$

or

$$y_7 - y_6 = \frac{\Delta P}{2\alpha_1}. \tag{4.9}$$

Since we know that $y_6 \cdot y_7 \equiv 0$, $2\alpha_1 < 0$, $y_6 \leq 0$, $y_7 \geq 0$, $\Delta P > 0$ we determine that

$$y_6 = \left|\frac{\Delta P}{2\alpha_1}\right|. \tag{4.10}$$

Figure 4.2 illustrates the above formulations. As it can be seen, if the optimal value of y_1 (denoted by y_1^*) is below the quantity floor, then the government must make sales from existing government stocks so that the available quantity achieves the quantity floor. In other words

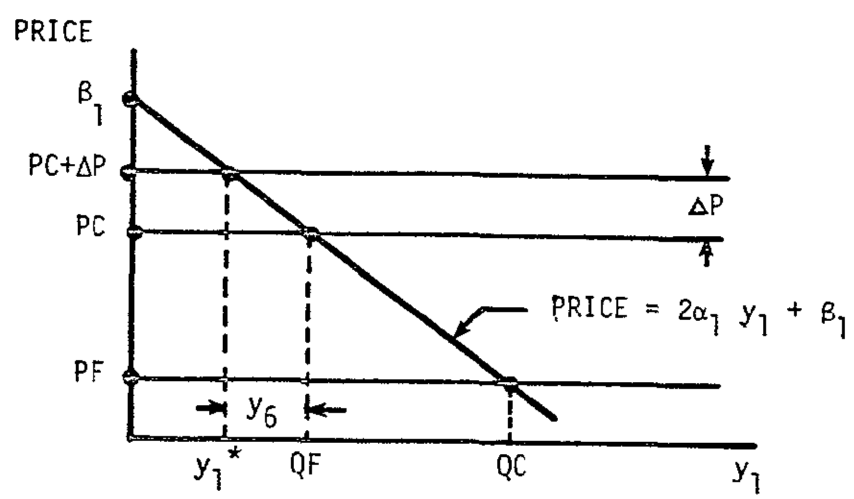

FIGURE 4.2 ILLUSTRATION OF A VIOLATION OF THE PRICE CEILING

$$y_1^* + y_6 = QF$$

or

$$y_6 = QF - y_1^* \tag{4.11}$$

whenever the price ceiling is violated.

When the price floor is violated, an identical argument is applicable and the government purchase is defined as

$$y_7 = \left|\frac{P}{2\alpha_1}\right| \tag{4.12}$$

or

$$y_7 = y_1^* - QC. \tag{4.13}$$

Since y_6 and y_7 are the amount of government action required to maintain the price bounds, certain straightforward conditions must be placed upon them in order to insure that they are reasonable.

The first constraint is that the government sales are less than or equal to the existing government stocks. In order to implement this constraint, an exogenous variable must be added to the system as

$0 \leq X_5$ = level of government stocks.

We assume that if sufficient government stocks are not available to reduce the price of the price ceiling, then all existing stocks are sold and the price constraint remains violated. A sufficiently high initial level of government stocks will insure that the above infeasibility never occurs. Thus we want

$$y_6 \leq X_5 . \tag{4.14}$$

The second constraint is that government purchases cannot exceed the existing supply on the marketplace. Thus,

$$y_7 \leq X_1 - y_1 - y_2$$

or

$$y_1 + y_7 \leq X_1 - y_2$$

where y_2 is the export of wheat. In other words, government purchases plus consumption has to be less than the existing supply minus exports.

A state transformation is required for the exogenous X_5 and is defined as

$$X_5^{t+1} = X_5^{t} - y_6^{t+1} + y_7^{t+1}$$

or the government stocks of the current period are the government stocks of the previous period minus government sales plus government purchases. Since the government transactions affect the existing supply, the state transformation for X_1 must be altered as follows

$$x_1^{t+1} = X_1^{t} - y_1^{t+1} - y_2^{t+1} + y_6^{t+1} - y_7^{t+1} .$$

In other words, all government transactions come from and go to the existing supply. This is the only way in which the government intervention simulation implementation impacts the existing model.

For an international grain reserve, the implementation requires four distinct transactions.

1. Initial purchase of stocks
2. Purchases to maintain price floor
3. Sales to maintain price ceiling
4. Disposition of remaining stock at end of simulation.

The assumptions of the transactions are as follows. The initial purchase of grain reserve stocks is made at the initial period at the current market prices prevailing in both regions. The initial level of stocks is an input parameter which can be varied for sensitivity analysis. All purchases are made at the price floor. All sales are made at the price ceiling. At the end of the simulation, the remaining stocks are sold at the existing market prices prevailing in the region where the stocks are held. These costs are tracked over the period of the simulation and a total discounted present value of grain reserve costs is the primary output. Since the present value includes the purchase of the stock in the initial period and the future transactions are discounted, the present value will always be negative.

5. RESULTS OF SIMULATION IMPLEMENTATION

The International Food Fund (IFF) simulation model as described in Section 1.5 was implemented in FORTRAN on the Princeton University IBM 370/158 computer. The converged value function coefficients were obtained from the ECON Integrated Model. Several parametric runs of the programs were made to determine the overall behavior of the IFF simulation model. The parameters of most interest for analyses were the respective values for the price bands for the United States and ROW and the starting IFF stocks. The simulation was run through a 50-year period for several alternative combinations of price bands and starting stocks with both current and LANDSAT information systems. The information systems were given the same performance measures used in previous ECON benefit/cost studies [15]. LANDSAT performance evaluation is based on the General Electric Sigma Squared Study [29].

The principal outputs from the model were the present value of the IFF cost to maintain the stocks, and the present value of world (U.S. and ROW) benefits from the fund. Since the costs of the transactions were discounted, the dominant cost appearing in the present value calculation in case of significant starting stocks was the purchase of an initial inventory. This initial inventory was set at such a level that the fund would only just not run out at any time in the 50 years so as to fulfill the policy goals at minimum cost. Final residual stocks in IFF were sold off at prevailing market price after 50 years. The IFF initial purchases and final sales were not allowed to affect prices. While somewhat unrealistic, this assumption has very little effect on the character of the results.

The length of the simulation runs--50 years--was chosen after determination of the sensitivity of the present value of the discounted stream of benefits to run length. It was found that significant changes in this quantity occurred after 25 years, but that it was insensitive to increased run length beyond 50 years. As a consequence of the long horizon implicit in our policy simulation methodology, the required initial IFF inventory was rather large. In future work it would be desirable to simulate a more conservative policy; for example, one which allows replenishment of the IFF inventory <u>outside of the price band</u> every five or ten years in addition to IFF purchases made solely to support prices at the price floor. The one case of a high ($160/metric ton) ROW price support level had to be rejected from the subsequent benefit analysis due to its anomalous cost indications.

The set of results used the converged coefficients of the value function with a 15 percent rate of discount. This rate was selected to represent inventory carrying charges--approximately 5 percent for storage costs and 10 percent for interest. Table 5.1 presents a comparison of the costs of an IFF for several cases of starting IFF inventory and ROW price floor, all other policy variables being held constant.

Notice that in all of the cases except one,* the cost to the fund of maintaining the price stabilization policy is less with the LANDSAT information system than with the current information system. The main reason for this difference is that, with better information, the IFF can start with smaller stocks. Since these starting stocks have to be purchased at prevailing market prices, the IFF costs are less. In reality, a stockpile would be built up gradually out of surplus

*The anomaly is due to the fact that this particular policy results in a long-term <u>upward</u> trend in IFF stocks which is counter to the policy goal of using the reserve to benefit consumers and producers.

TABLE 5.1 COMPARISON OF AN IFF WITH CROP INFORMATION OBTAINED FROM LANDSAT INFORMATION SYSTEMS: INITIAL STOCKS (REQUIRED) AND PRESENT VALUE COSTS WITH TWO HORIZONS

ROW PRICE BAND	CURRENT		LANDSAT		DIFFERENCE	
25-YEAR HORIZON	INITIAL STOCKS MMT	PRESENT VALUE COSTS $	INITIAL STOCKS MMT	PRESENT VALUE COSTS $	INITIAL STOCKS MMT	PRESENT VALUE COSTS $
$150-258	43.8	2.95B	23.6	2.15B	20.2	0.80B
$155-258	27.1	1.50B	6.5	0.51B	20.6	0.99B
$160-220	20.4	1.40B	0.0	0.33B	20.4	1.05B
50-YEAR HORIZON						
$150-258	90.7	9.22B	42.2	4.59B	48.5	4.63B
$155-258	52.1	4.80B	11.7	1.12B	40.4	3.68B
$160-220	11.1	3.80B	0.0	6.59B	11.1	-2.79B*

*NOTE: THE ANOMALOUS INDICATION OF GREATER IFF COSTS WITH IMPROVED CROP INFORMATION IS NOT USED IN SUBSEQUENT ANALYSES.

private or U.S. government stocks, thus reducing both IFF costs and market impact.

Figure 5.1 illustrates the U.S. price series (without price constraints) and Figure 5.2 illustrates the ROW price series for both current and satellite information systems. For both information systems, the price peaks are very sharp and there is no comparable valley to lower prices. Since high prices occur with undersupply and low prices occur with oversupply, these results illustrate that the model optimization tends to reduce the possibility of oversupply. In reality, large surpluses do occur due to bumper crops being harvested in many parts of the world. The plot also shows that the domestic price in the satellite case tends to be, in general, higher with approximately the same number of peaks. The major difference in the two cases is that the magnitude of the peaks in the satellite case is significantly larger than the magnitude of the peaks in the current information case for U.S. prices and the reverse for ROW prices.

The second principal output variable is the per period IFF transactions and the level of IFF stocks. For the sake of fulfilling the policy goals over many years, the level of IFF stocks should remain approximately stable; that is to say that there should be no long-term trend. If there were a generally increasing trend to the level of IFF stocks, the average annual IFF purchases would be generally higher than the average annual IFF sales. This implies that the price floor has been violated more often or with greater magnitude than has the price ceiling. To eliminate such a trend, the price levels of the floor and/or ceiling can be adjusted until an equilibrium is attained in the level of IFF stocks.

Table 5.2 presents some statistics pertaining to the level of IFF stocks and transactions for the 50-year simulation runs. The level of initial IFF stocks was chosen so as just to avoid any stock-out during the 50-year period of the

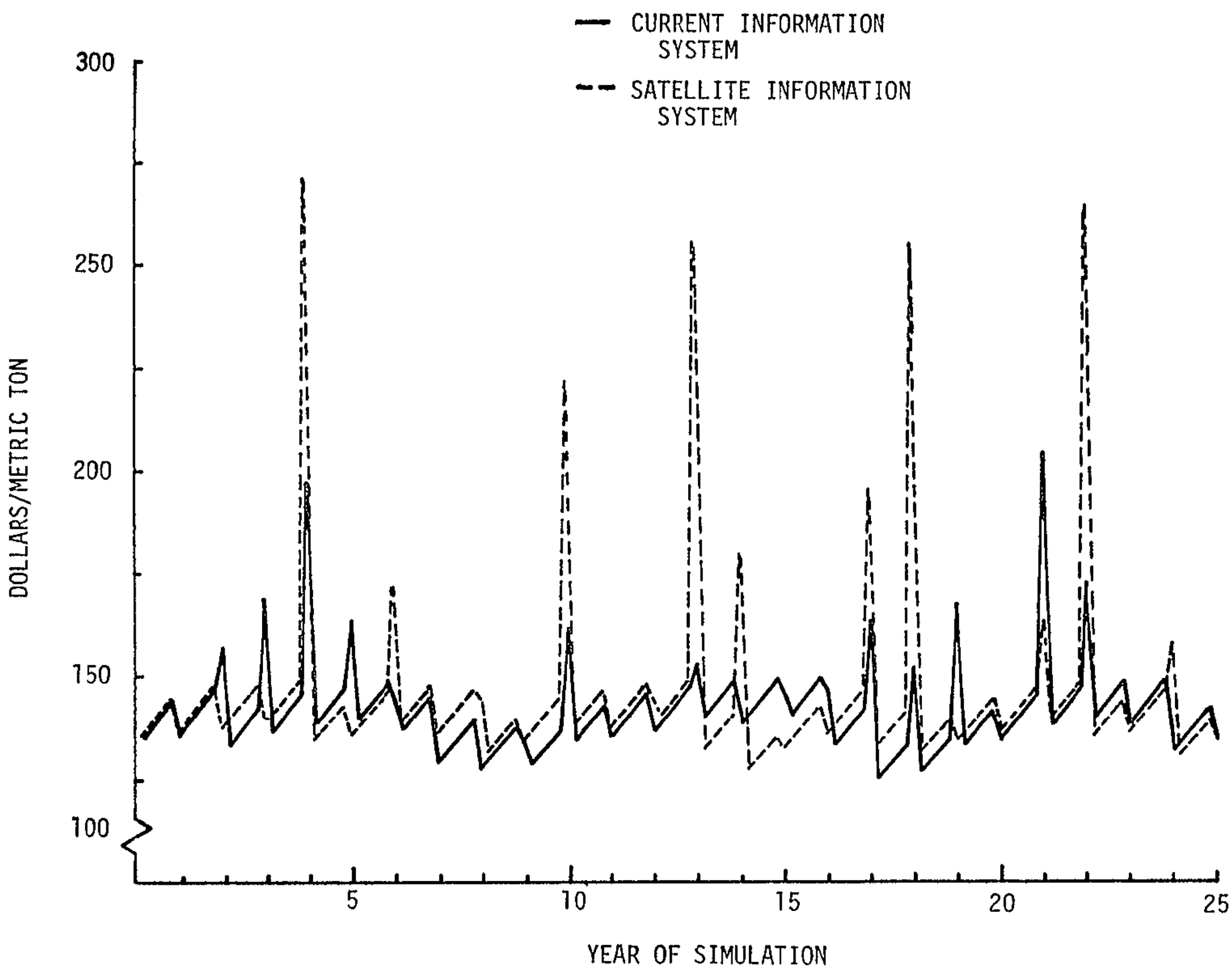

FIGURE 5.1 SIMULATION MODEL DOMESTIC PRICE SERIES CURRENT VS. SATELLITE INFORMATION SYSTEM WITHOUT PRICE CONSTRAINTS

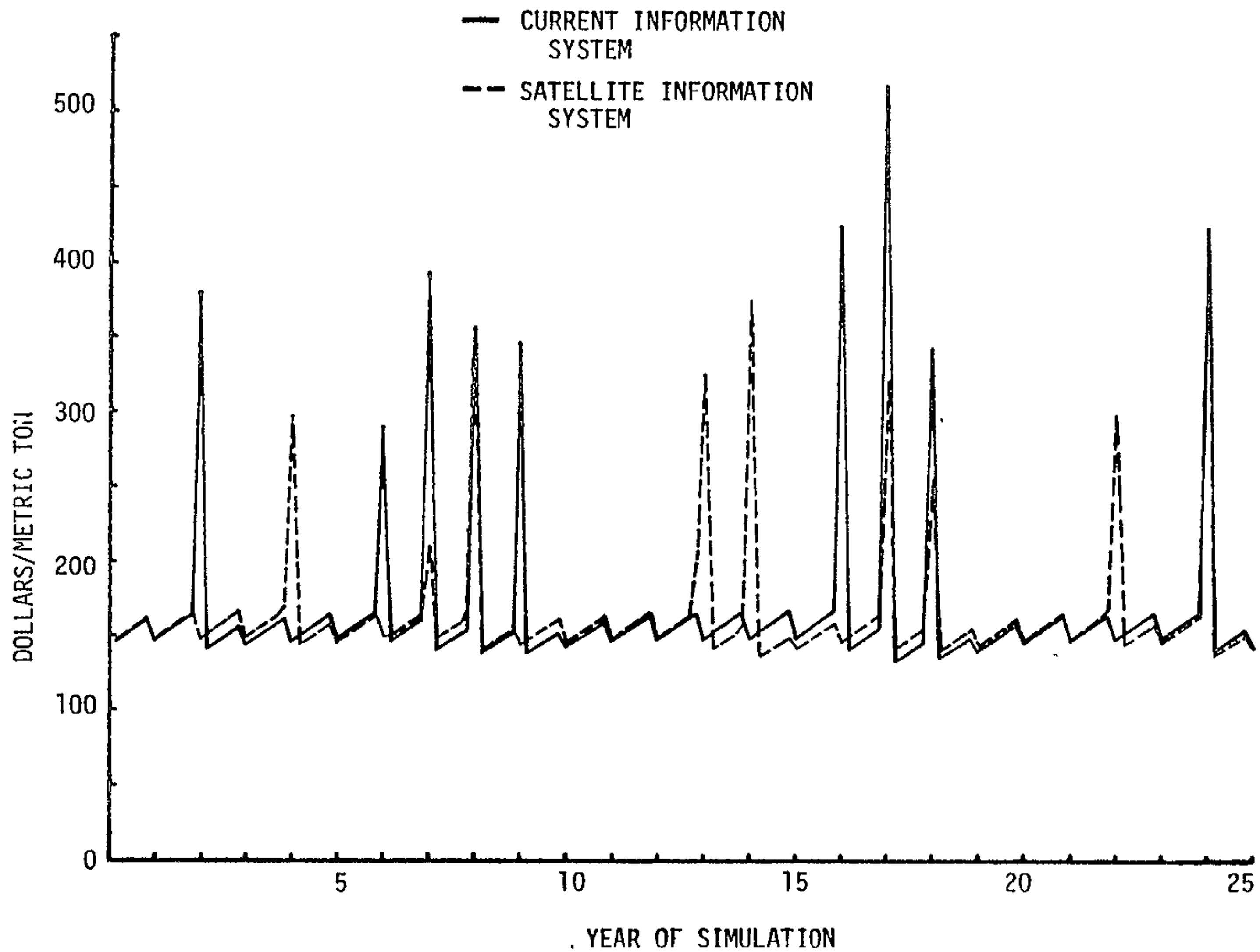

FIGURE 5.2 ROW PRICE SERIES FOR 25-YEAR SIMULATIONS WITH CURRENT AND SATELLITE INFORMATION SYSTEMS

TABLE 5.2 DETAILS OF THE INTERNATIONAL GRAIN RESERVE POLICY SIMULATION (50 YEARS)

	ROW PRICE BAND*			
	$150-250/MT		$155-250/MT	
	CURRENT	LANDSAT	CURRENT	LANDSAT
INITIAL STOCK	90.71 MMT	42.19 MMT	52.12 MMT	11.68 MMT
FINAL STOCK	7.86 MMT	2.82 MMT	9.46 MMT	9.57 MMT
AVE. U.S. PURCHASE	0.00 MMT	0.04 MMT	0.00 MMT	0.04 MMT
AVE. U.S. SALE	0.05 MMT	0.03 MMT	0.05 MMT	0.03 MMT
AVE. ROW PURCHASE	0.94 MMT	0.48 MMT	0.95 MMT	0.49 MMT
AVE. ROW SALE	0.33 MMT	0.23 MMT	0.62 MMT	0.49 MMT
CONSUMER GAINS (LOSSES)				
U.S.	$ -28 M	$+1.0 M	$ -27 M	$+2.5 M
ROW	$+886 M	$+260 M	$+201 M	$-306 M

*U.S. PRICE BAND = $140-220/MT IN BOTH CASES. FIFTEEN PERCENT P.A. DISCOUNTING.

simulation. From Table 5.2 we learn that the average U.S. net transactions of the IFF are always larger and average ROW net transactions are always smaller (algebraically) with satellite as compared with current information systems. Most notable of the results of these policy simulations, however, are the dramatic effects of (1) the ROW price floor, and (2) the improved crop information system on the required size of the initial IFF stocks. By increasing the price floor, one induces ROW to supply more of its surplus wheat to the IFF, thus making it less dependent on the initial stocks. ROW purchases from the fund are not much affected by this variable. On the other hand, the improved information system drastically reduces the ROW need for IFF buffer stocks; both the size of the starting IFF stocks and the average ROW transactions are reduced. This result reflects in a quantitative way the trade-off between wheat buffer stocks and crop information which has been discussed speculatively in the past.

The extent to which the fund creates benefits depends on the rules of its usage. In analyzing the benefits, it is important to remember that the IFF must purchase wheat from the regional markets when prices are low as well as selling wheat in times of shortage at higher prices. When wheat is purchased by the fund, prices are driven up, creating a consumer disbenefit. Thus, the benefits of IFF releases at relatively high prices are offset, to some extent, by the disbenefits caused by IFF acquisitions. If the fund is poorly managed, or if the price bands are not chosen judiciously, the net effect may be an economic loss rather than a benefit. Table 5.3 presents the economic costs and benefits of an IFF with and without LANDSAT. The economic effects of improved information due to LANDSAT are clearly to reduce both the required starting inventory and the cost of the IFF. At the same time, benefits are also reduced. The net effect is an economic gain for the world if the policy is implemented with LANDSAT rather

TABLE 5.3 COSTS AND BENEFITS OF AN INTERNATIONAL GRAIN RESERVE WITH AND WITHOUT LANDSAT (50-YEAR SIMULATION)

	ROW PRICE BAND*					
	$150-250/MT			$155-250/MT		
	INITIAL STOCK (MMT)	ANNUALIZED COSTS ($)	ANNUALIZED BENEFITS ($)	INITIAL STOCK (MMT)	ANNUALIZED COSTS ($)	ANNUALIZED BENEFITS ($)
CURRENT	90	-1204M	703M	52	-627M	535M
LANDSAT	42	- 600M	243M	12	-147M	101M
DIFFERENCE	48	604M	-460M	40	480M	-434M
NET BENEFIT OF LANDSAT SYSTEM		144M			46M	

*NOTE: U.S. PRICES ARE MAINTAINED WITHIN $140-220/MT.

than with current crop information standards. This gain is $100 million per year larger using the $150 ROW price support than it is with the $155 price support, thus indicating sensitivity of the economic results to the choice of the IFF policy rules.

In order to analyze the effect on fund costs and benefits of the improved (LANDSAT) information system, we select the most "reasonable" of the simulation cases, which is the first case (price band $150 to $250 for ROW). The annualized 50-year IFF costs are reduced by 50 percent as a result of the LANDSAT crop information, largely due to the much lower required starting stocks. However, at the same time, consumer benefits throughout the world are reduced by 70 percent, most of this reduction occurring in the ROW. The improvement of wheat forecasts--specifically the reduction of forecast mean square error--in our model achieves some of the purposes of the fund, and hence reduces the fund's potential for benefiting consumers of wheat.

Comparing IFF costs with benefits for the same case ($150 to $250), we find that, under current information, the benefit-to-cost ratio is 0.58, while under improved information this ratio drops to 0.41. Using this criterion for deciding whether or not to create a fund, it appears that one would prefer not to create the IFF at all on strictly economic grounds regardless of the quality of the information system. However, if the decision to create an IFF has already been made on other grounds, the implementation of the LANDSAT information system generates substantial cost savings which can be translated into a net economic benefit with suitable choice of the policy rules.

6. REFERENCES

1. Eaton, David J. and W. Scott Steele, "Analysis of Grain Reserves: A Proceedings," USDA Economic Research Service, ERS-634, Washington, District of Columbia, August 1976.

2. Tweeten, Kalbfleisch and Lu, "An Economic Analysis of Carryover Policies for the United States Wheat Industry," Oklahoma State University Technical Bulletin T-132, October 1971.

3. Steele, W. Scott, "Discussion of Alternative Grain Reserve Policies," FDCU Working Paper, FDCD, USA, January 31, 1974.

4. Sharples, Jerry A. and Rodney Walker, "Reserve Stocks of Grain," Research Status Report No. 1, September 1974.

5. ____, "Analysis of Wheat Loan Rates and Target Prices Using a Wheat Reserve Stocks Simulation Model," Research Status Report No. 2, May 1975.

6. Sumner, Daniel and D. Gale Johnson, "Determination of Optimal Grain Carryovers," University of Chicago, Office of Agricultural Economic Research, Paper No. 74:12, October 31, 1974.

7. Ray, Daryll E., James W. Richardson and Glenn J. Collins, "A Simulation Analysis of a Reserve Stock Management Policy for Feed Grains and Wheat," Oklahoma Agricultural Experiment Station Journal, Article J-2823.

8. Reutlinger, Shlomo, "Evaluating Wheat Buffer Stocks," American Journal of Agricultural Economics, Vol. 58, No. 1, February 1976.

9. Helmberger, Peter and Rob Weaver, "Welfare Implications of Commodity Storage Under Uncertainty," American Journal of Agricultural Economics, Vol. 59, No. 4, November 1977.

10. Samuelson, Paul, "The Consumer Does Benefit from Feasible Price Stability," Quarterly Journal of Economics, August 1972.

11. Hayami, Y. and W. Peterson, "Social Returns to Public Information Services: Statistical Reporting of U.S. Farm Commodities," American Economic Review, LXII, 1972.

12. Subotnick, A. and J. P. Houck, "Welfare Implications of Stabilizing Consumption and Production," American Journal of Agricultural Economics, Vol. 58, No. 1, February 1976.

13. Bradford, David F. and Harry H. Kelejian, "The Economic Value of Remote Sensing of Earth Resources from Space: An ERTS Overview and the Value of Continuity of Service," Vol. III, Part II, ECON, Inc., Princeton, New Jersey, Report No. 74-2002-10, December 31, 1974.

14. Heiss, K. P., "United States Benefits of Improved Worldwide Wheat Crop Information from a LANDSAT System," ECON, Inc., Princeton, New Jersey, Report No. 76-122-1B, August 31, 1975.

15. Andrews, John, "Economic Benefits of Improved Information on Worldwide Crop Production," ECON, Inc., Princeton, New Jersey, Report No. 76-243-1A, April 15, 1977.

16. Massell, Benton, "Price Stabilization and Welfare," Quarterly Journal of Economics, Vol. 73, No. 2, May 1969.

17. Stein, John P. and Rodney T. Smith, "The Economics of United States Grain Stockpiling," RAND Report No. R-1861, March 1977.

18. Stein, John P., Emmett Keeler and Rodney T. Smith, "U.S. Grain Reserves Policy: Objectives, Costs and Distribution of Benefits," RAND Report No. R-2087-RC, February 1977.

19. Hillman, Jimmye, D. Gale Johnson and Roger Gray, "Food Reserve Policies for World Food Security," U.N. Food and Agriculture Organization, Rome, 1975.

20. Gustafson, Robert L., "Carryover Levels for Grains: A Method for Determining Amounts that Are Optimal Under Specified Conditions," USDA Technical Bulletin No. 1178, 1958.

21. Johnson, D. Gale and David Sumner, "An Optimization Approach to Grain Reserves for Developing Countries," Analysis of Grain Reserves: A Proceedings, compiled by David J. Eaton and W. Scott Steele, USDA-ERS, August 1976.

22. Pearce, William R., "Groping for a New World Wheat Pact," New York Times, September 11, 1977.

23. Just, Richard E., Ernst Lutz, Andrew Schwitz and Stephen Turnovsky, "The Distribution of Welfare Gains From International Price Stabilization Under Distortions," American Journal of Agricultural Economics, Vol. 59, No. 4, November 1977.

24. Konandreas, P., B. Huddleston and V. Ramangkura, Food Security: An Insurance Approach, International Food Policy Research Institute, Research Report 4, September 1978.

25. Bigman, David and Shlomo Reutlinger, "National and International Policies Toward Food Security and Price Stabilization," paper presented at the American Economic Association Meeting, Chicago, August 1978.

26. Heiss, K. P. and R. A. Fish, "A Cost-Benefit Evaluation of the LANDSAT Follow-On Program," ECON, Inc., Princeton, New Jersey, Report No. 76-102-3, September 15, 1976.

27. Thurow, Lester C., "ECON Evaluation of LANDSAT System," memorandum commissioned by Battelle under contract to NASA, March 8, 1976. (This memorandum is also reproduced in: "Sensitivity Analysis of the ECON Agriculture Information Models," ECON Report No. 76-102-1A, prepared for NASA under Contract No. NASW-2558, August 31, 1976.)

28. Taylor, C. Robert and Hovav Talpaz, "Approximately Optimal Carryover Levels for Wheat in the United States," American Journal of Agricultural Economics, Vol. 61, No. 1, February 1979.

29. General Electric Space Division, "Sigma Squared Study," prepared for NASA Goddard Space Flight Center under Contract No. NAS5-23412, Mod. 30, 1977.

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